Interaction

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Introduction

- Many causal questions are about the effects of two or more simultaneous treatments.
- This chapter provides a formal definition of interaction between two treatments , both counterfactual framework and the sufficient-component-cause framework.

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Interaction between two treatments

- $Y = 1$(die), $Y = 0$(survive)
- $A = 1$(heart transplant), $A = 0$(no heart transplant)
- $E = 1$(vitamin), $E = 0$(no vitamin)

Definition

We say that there is interaction between A and E on the additive scale in the population if

$$P(Y^{a=1,e=1} = 1) - P(Y^{a=0,e=1} = 1) \neq P(Y^{a=1,e=0} = 1) - P(Y^{a=0,e=0} = 1)$$

- Difference between interaction and effect modification.
- The concept of effect modification refers to the causal effect of A, not to the causal effect of V.

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Identifying interaction

- If E is randomly assigned, $P(Y^{a=1,e=1} = 1) = P(Y^{a=1} = 1|E = 1)$.
- The interaction between A and E is rewritten by

$$P(Y^{a=1} = 1|E = 1) - P(Y^{a=0} = 1|E = 1)$$
$$\neq P(Y^{a=1} = 1|E = 0) - P(Y^{a=0} = 1|E = 0)$$

Counterfactual response types and interaction

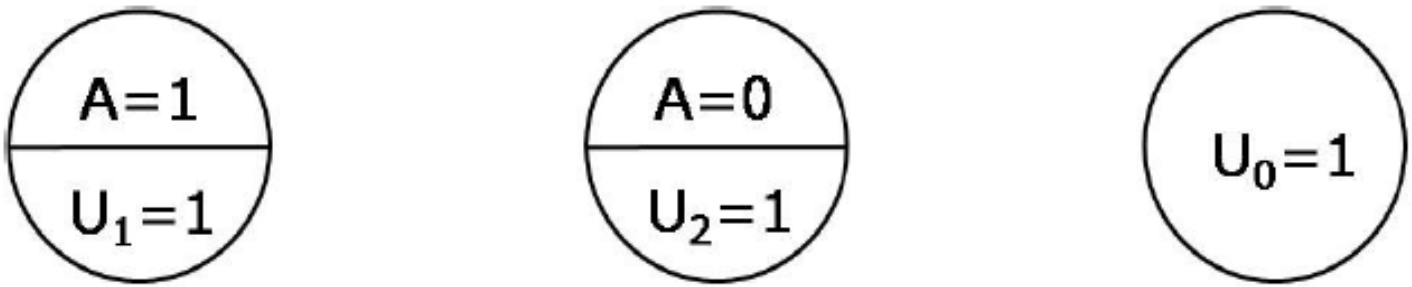
Table 5.2

	$Y^{a,e}$ for each a, e value			
Type	$1,1$	$0,1$	$1,0$	$0,0$
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	0	1	1	1
10	0	1	1	0
11	0	1	0	1
12	0	1	0	0
13	0	0	1	1
14	0	0	1	0
15	0	0	0	1
16	0	0	0	0

- Type 1, 4,6, 11, 12, 13, 16 : No interaction between A and E

Sufficient Cause

- Take those who were treated.
- Some died; others survived.
  1. Heart transplant($A = 1$) only results in death in individuals allergic to anesthesia($U_1 = 1$). ($A = 1, U_1 = 1 \rightarrow Y = 1$)
  2. No heart transplant($A = 0$) only results in death if individuals have ans ejection fraction less than 20%($U_2 = 1$) ($A = 0, U_2 = 1 \rightarrow Y = 1$)
  3. All individuals with pancreatic cancer($U_0 = 1$) at the start of the study will die. ($U_0 = 1 \rightarrow Y = 1$)



Sufficient cause interaction

- A sufficient cause interaction between A and E exists in the population if A and E occur together in a sufficient cause.

Examples (sufficient cause interaction)

- $A = 1, E = 1 \rightarrow Y = 1$
- $A = 1, E = 0 \rightarrow Y = 0$
- $A = 0, E = 1 \rightarrow Y = 0$

- Sufficient cause interaction between A and E is **synergistic** if $(A = 1, E = 1)$ are present in the same sufficient cause.
- Sufficient cause interaction between A and E is **antagonistic** if $(A = 1, E = 0)$ or $(A = 0, E = 1)$ are present in the same sufficient cause.